Exceptional points due to asymmetric perturbations in deformed optical microdisk cavities

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We study exceptional points (EPs) due to asymmetric perturbations in deformed optical microdisk cavities.

Unlike Hermitian systems ($H^\dagger = H$), non-Hermitian systems ($H^\dagger \neq H$) can exhibit coalescence of both eigenvalues and eigenstates at degeneracy points known as EPs, where $\psi_{\mathrm{i}} = \psi_{\mathrm{j}}$, and $E_{\mathrm{i}} = E_{\mathrm{j}}$ [1, 2]. These points, characterized by branch-point topology on the Riemann surface, yield highly sensitive spectral responses with frequency splitting $\Delta\omega \sim \varepsilon^{1/N}$ under small perturbations $\varepsilon \ll 1$, an effect valuable for optical sensing [3, 4].

We present a systematic formation of EPs in the parameter space and demonstrate that the sensitivity of EP-based sensors can be maximized by carefully examining the continuous formation of EPs. To demonstrate this, we design an elliptic-shaped microdisk using three deformation parameters (ε_1, ε_2, ε_3):

$$r(\varepsilon_1, \varepsilon_2, \varepsilon_3; \theta) = \rho(\varepsilon_1, \varepsilon_2, \varepsilon_3)\left[\frac{1}{\sqrt{1-\varepsilon_1^2\cos^2\theta}} + \frac{\varepsilon_2^2\cos(\theta+\pi\varepsilon_3)}{\sqrt{1-\varepsilon_1^2\cos^2\theta}}\right].$$

This geometry enables systematic control over mode coupling, allowing EPs to form continuously from a pair of even and odd standing-wave modes with identical radial and angular number (*l, m*). As the deformation parameters are carefully tuned, this pair coalesces into chiral EPs that manifest as purely traveling waves propagating in either the clockwise (CW) or counterclockwise (CCW) direction under symmetry breaking. Using an effective non-Hermitian Hamiltonian, we analyze the generation of chiral EP modes $H_{\mathrm{EP}} = (E_{\mathrm{EP}}, 0; V_{\mathrm{EP}}, E_{\mathrm{EP}})$ and further characterize how the frequency splittings in the vicinity of EPs scale with both intrinsic coupling strength and externally applied perturbations $H = H_{\mathrm{EP}} + H_{\mathrm{Pert}}$.

This study provides new insights into the controlled design of EP-based microdisk cavities and we believe our results will contribute to the development of high-resolution EP-based optical sensors for cutting-edge photonic applications.

References

1. W. D. Heiss, "Repulsion of resonance states and exceptional points," Phys. Rev. E 61, 929 (2000).
2. M. V. Berry, "Physics of nonhermitian degeneracies," Czech. J. Phys. 54, 1039 (2004).
3. J. Wiersig, "Sensors operating at exceptional points: general theory," Phys. Rev. A 93, 033809 (2016).
4. W. Chen, Ş. K. Özdemir, G. Zhao, J. Wiersig, and L. Yang, "Exceptional points enhance sensing in an optical microcavity," Nature 548, 192 (2017).